The jamming perspective on wet foams

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Amorphous materials as diverse as foams, emulsions, colloidal suspensions and granular media can jam into a rigid, disordered state where they withstand finite shear stresses before yielding. The jamming transition has been studied extensively, in particular in computer simulations of frictionless, soft, purely repulsive spheres. Foams and emulsions are the closest realizations of this model, and in foams, the (un)jamming point corresponds to the wet limit, where the bubbles become spherical and just form contacts. Here we sketch the relevance of the jamming perspective for the geometry, mechanics and rheology of foams.

1. Introduction

Liquid foams and emulsions are dispersions of gas bubbles or droplets in an immiscible (second) liquid phase, stabilized by surfactants. Phenomenologically, these materials exhibit plastic flow under large stresses, but deform elastically under smaller stresses—as is the case for many other disordered soft materials, their macroscopic response to mechanical perturbations is a complex mix of elastic, plastic and viscous effects.1

How should we think about the mechanics of such materials, and more specifically, about the mechanics of foams? More precisely, how are the laws that govern the local dynamics of bubbles related to the macroscopic, collective behavior of a foam? One extreme point of view would be to assume that the complexity at the global level—say, nonlinear rheology—follows from a similar complexity at the local level, such as similarly nonlinear viscous bubble interactions.2 Another extreme point of view would be that none of the local details matter and that the global behavior of disordered media, and foams in particular, is set by “universal” collective mechanisms3,4—so that different local viscous laws would lead to the same macroscopic rheology.

Here we show that local details do matter, while at the same time qualitatively new behavior emerges at the global level. We suggest that the mechanisms that connect the micro- and macro-world are universal, though. We will illustrate our point of view by discussing recent experimental and numerical work on foams, and by making explicit contact with the jamming framework that has been emerging in recent years.5–7

We first briefly review insights into the crucial role of disorder in foam-like systems, stressing the breakdown of the affine assumption, and show that crucial theoretical jamming predictions have been observed recently in experiments on 2D foams. Then we discuss experiments on and models for the rheology of foams and other related soft disordered media.

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2. Jamming, disorder and non-affinity

The gas fraction $\phi$ clearly plays a crucial role in determining foam’s structure and rigidity, and some of the earliest studies that consider the loss of rigidity in disordered media—what we now call unjamming—concern foams and emulsions with increasing wetness.\cite{whitelam2005unjamming, katgert2005unjamming, muller2007unjamming} The (un)jamming scenario for foams is illustrated in Fig. 1. When the gas fraction approaches unity, the foam is called dry. Macroscopic deformations of such foams cause stretching of the liquid films that provide restoring forces—dry foams are jammed. When the gas fraction is lowered and the foam becomes wetter, the gas bubbles become increasingly spherical, and the foam loses rigidity for some critical gas fraction $\phi_c$ where the bubbles lose contact (Fig. 1). The unjamming transition is thus governed by the gas fraction, which typically is seen as a material parameter. For emulsions essentially the same scenario arises.\cite{bechinger2006structure}

As the interactions between bubbles are dominantly repulsive and viscous, static foams are similar to packings of frictionless soft spheres—precisely the models studied extensively in jamming.\cite{lehner1999capture} In real foams, gravity (which causes drainage) and gas diffusion (which causes coarsening) play a role, although these effects can be minimized by studying quasi-2D foams and using inert gases.\cite{robert2007unjamming, martinez2008unjamming, van2010jamming}

![Diagram of foam states](image)

**Fig. 1**: Topview of 2D foams, consisting of a mix of 2 and 3 mm bubbles trapped below a top plate. At low packing fractions (left), the bubbles do not form contacts and the materials are in a mechanical vacuum state. At high packing fractions (right), the bubbles are squeezed together and form a jammed, rigid state. At intermediate packing fractions, the bubbles just touch and form a marginal state.

**Fig. 2**: Average contact number versus $\phi$ for experimental bidisperse foams: grey dots indicate data for each individual realization and black circles indicate averages for each globally set packing fraction. The solid red line is a square root fit to the data (see the text). The inset shows the data plotted versus the experimentally determined packing fraction $\phi_{exp}$. The fit has a power law exponent of 0.70. Data from the work of Katgert \textit{et al.}\cite{katgert2005unjamming}

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**Disordered geometry**

How crucial is disorder? In ordered, “crystalline” foams such as two-dimensional hexagonal packings of monodisperse bubbles (“liquid honeycombs”),\cite{whitelam2005unjamming, katgert2005unjamming, muller2007unjamming} global deformations translate into a homogeneous local deformation field, as all the cells deform equally. In this case the bubbles lose contact at the critical density $\phi_c$ equal to $\frac{\pi}{2\sqrt{3}} \approx 0.9069$, and the average number of contacting neighbors per bubble, $z$, remains constant at 6 in the jammed regime. Similar results can be obtained for three-dimensional ordered foams, where $\phi_c$ is given by the packing density of the HCP lattice $\frac{\pi}{3\sqrt{2}} \approx 0.7405$.

Disordered foams are, however, very different. Experiments and simulations clearly find that the critical packing fraction is substantially lower, around 84% in 2D and 64% in 3D.\cite{bechinger2006structure, roux2007unjamming, van2010jamming} Moreover, simulations have revealed that the contact number varies smoothly with the packing fraction,\cite{whitelam2005unjamming, katgert2005unjamming, muller2007unjamming} in 2D, the contact number in foams ranges from 6 in the dry limit and reaches the marginal, or isostatic value, $z_c = 4$ at the unjamming point—in addition, the excess coordination $z - z_c$ grows as a square root with $\phi - \phi_c$.

These numerical predictions have recently been confirmed in experiments on disordered monolayers of bubbles floating on the surface of a soapy solution and bound on the top by a well-leveled glass plate.\cite{robert2007unjamming, martinez2008unjamming, van2010jamming} The average contact number $z$ and the packing fraction can then be determined by image analysis. As shown in Fig. 2, the contact number tends to $z = 6$ for high packing fractions,\cite{robert2007unjamming} whereas the average contact number decreases as a square root, ultimately reaching $z_c = 4$ when the (2D) packing fraction is reduced to a critical value around $\phi = 0.84$.

Moreover, the variation of $z$ with $\phi$ is similar to a square root and can be fitted well by a power law fit of the form...
\[ z = 4 + z_0(\phi - \phi_c)^{\alpha}, \] with best fit values in remarkable agreement with numerical predictions. It should be noted that in simulation studies the packing fraction is often calculated by counting the overlaps between bubbles twice (allowing packing fractions exceeding one), and the experimentally determined packing fractions have to be converted to the corresponding theoretical packing fractions in order to reach good agreement with prior simulations (see the inset of Fig. 2). We also note that this square root scaling has recently been observed in 3D emulsions.

**Mechanics of disordered foams**

Even more crucial are the differences between the mechanical response of ordered and disordered foams. For liquid honeycombs the yield stress and shear modulus remain finite and jump to zero precisely at \( \phi_c \). In contrast, for disordered foams and emulsions they vanish smoothly when the critical packing fraction is reached. Early evidence comes from measurements in polydisperse emulsions by Princen, who observed a substantial lowering of the shear modulus when \( \phi \) is lowered. Later measurements of the shear modulus and osmotic pressure of compressed disordered emulsions and foams found similar behavior for the loss of rigidity. There is also ample numerical evidence for the smooth vanishing of the shear modulus and yield stress in simulations, although we note that the agreement between experiments and numerics is qualitative rather than quantitative.

**Non-affine response**

The crucial ingredient to understand the role of disorder for the mechanical response is the non-affine nature of the deformations of disordered media (Fig. 3). In an affine deformation all the local deformations simply follow the global deformation, implying that the local motion of the particles is as if they were pinned to a rubber sheet. However, in a disordered system such as a foam, where particles interact with their neighbors, the local, disordered environment sets the motion. The key observation is that materials far away from the jamming point (Fig. 3a) exhibit deformations that are close to affine (Fig. 3b), while materials closer to the jamming point (Fig. 3c) exhibit deformations that become increasingly disordered and non-affine (Fig. 3d).

The role of this non-affinity is crucial for the mechanics of jammed materials such as foams. The scenario is as follows: far away from jamming, the local deformations are similar to the global ones, and for linear interactions with a local spring constant \( k \) (set by the surface tension), one concludes that the shear \( (G) \) and bulk \( (K) \) moduli are both on the order of \( k \). Closer to jamming, however, the number of contacts per bubble drops, and bubbles are increasingly free to deviate from the affine field so as to minimize the changes in elastic energy, and, as a consequence, the shear modulus \( G \) (but not the bulk modulus) becomes significantly lower than would be expected from an affine assumption. At the jamming point the shear modulus vanishes for harmonically interacting particles—clearly, near jamming, naively extrapolating from the local properties is an exceedingly poor way to predict the global behavior. More precisely, \( K \propto k \), whereas \( G \propto (z - z_c) k \), which also holds for nonlinear interactions such Hertzian interactions. This illustrates our earlier point: local interactions matter, but there are in addition universal and nontrivial mechanisms that translate between the local interactions and the global behavior.

**3. Flow of foams**

Slowly flowing foams, emulsions, suspensions and similar amorphous media exhibit a rich rheology—what roles do disorder and jamming play here? 2D foams are eminently suited to study such questions experimentally, as they allow simultaneous probing of the rheology and imaging of the bubble motion.

**Rheology: experiments**

The role of disorder can be probed by comparing the flow profiles and bubble trajectories of partially ordered, monodisperse foam and those of disordered, bidisperse foam (Fig. 4). In the monodisperse foam (Fig. 4a) the bubbles move past each other in a zipper-like fashion along the crystal planes of the hexagonal lattice formed by the bubbles—see tracks in Fig. 4b. The velocity profiles (Fig. 4b) are strongly localized and are independent of driving velocity. In disordered foams (Fig. 4c) the situation is vastly different: the bubble motion becomes disordered and organizes into the swirling, collective patterns often seen in materials near jamming (Fig. 4d). The velocity profiles now depend on the driving velocity, becoming increasingly delocalized as it decreases.

We note here that without the top plate, the flow profiles are essentially linear—the shear banding is due to drag with the
top plate. By balancing the averaged viscous friction between bubbles that move past each other and the viscous friction due to the bubbles moving below the glass plate (scaling with bubble velocity $v$ as $v^{2/3}$ — a classic result from Bretherton), the averaged interbubble friction can be deduced from the velocity profiles. Rate independent velocity profiles, as seen in the ordered foam, can only arise if the interbubble viscous forces scale as the bubble–top plate friction: $F_v \sim \Delta v^{2/3}$, where $\Delta v$ is the velocity difference between neighboring bubbles, a scaling verified by rheological measurements (Fig. 4c) — here the averaged and bubble–bubble interactions are similar.

The rate dependent profiles observed for the disordered foams imply that the exponents for the averaged bubble–bubble and bubble–top plate friction are different — which implies that the averaged and bubble–bubble interactions are different. In fact, one can fit all flow profiles to an average interbubble friction that scales as $\Delta v^{2/3}$, an exponent that can also be obtained by direct rheological measurements (Fig. 4f). We thus find that disorder changes the effective viscous friction between bubbles in a highly nontrivial way: the averaged viscous dissipation inside the foam is enhanced with respect to the ordered case, leading to less localized, rate dependent flows.

Rheology: bubble models

A similar mechanism has also been seen in numerical models for wet foams, which stems from the “bubble model” introduced in 1995 by Durian. In these models all bubbles are represented by soft spherical (disk-like in 2D) particles that interact only when in contact. Damping dominates inertia in foams, so the equations of motion are assumed to be overdamped, implying that at any instant the net elastic and viscous forces on each bubble balance.

To specify a particular variant of the bubble model one must describe the elastic and viscous forces between bubbles. Elastic forces can be generated by harmonic “one-sided springs”, see Fig. 5a, or nonlinear extensions of this equation:

$$f_{el} \sim b^\delta v_{el}.$$  

where $\delta$ is the dimensionless compression of the spring and $\alpha_{el} = 1 (3/2)$ corresponds to harmonic (Hertzian) forces. Similarly, it is natural to invoke a viscous force law in which the relative velocity of contacting bubbles is damped (Fig. 5b):

$$f_{visc} = b|\Delta v|^\alpha_{visc}$$

For most simulations $\alpha_{visc} = 1$, although real foams have $\alpha_{visc} < 1$, with the actual value depending on the surfactant used. Note that a mean field dissipation, which sacrifices realistic modeling for numerical convenience, is also frequently encountered in the literature — here each bubble experiences a damping force proportional to the difference between its instantaneous velocity and the mean velocity.

In steady flows the constitutive relation is qualitatively similar to the Herschel–Bulkley constitutive relation when the packing fraction is sufficiently large:

$$\sigma = \sigma_v + A \gamma^\beta$$

The exponent $\beta$ is typically less than one, a property known as “shear thinning.” Simulations find $\beta < \alpha_{visc}$, so the averaged bubble–bubble drag is different from the individual bubble–bubble drag, just as in the experiments.

Fluctuations

The nontrivial rheology of wet foams is closely related to the nature of the fluctuations. Without going into much detail, experiments on flows close to but above $\phi_c$ find that fluctuations increase as the flow rate decreases, i.e. as the critical jamming point is approached. Imagine you are given two movies of foam flows at two different flow rates — but you are not told which one is the fastest, nor do you know the frame rate of these movies. By adjusting their playback speed, you can make the average flow rate of the two movies equal — the data then show that fluctuations would be strongest in the movie.
corresponding to the slowest flow. In passing we note that this suggests that there is no well-defined quasistatic limit for particle trajectories in yield stress fluids: the slower the flow rate, the larger the fluctuations.

The growth of the fluctuations can be understood from a precise relationship between the fluctuations, rheology and dissipation. If one shears foam (or any other system) at a rate \( \dot{\gamma} \) and concomitant shear stress \( \sigma \), the product \( \sigma \dot{\gamma} \) sets the average rate of work done on the system. This work has to be dissipated, and for wet foams the only dissipative mechanism is the relative motion of neighboring bubbles. In the bubble model the viscous force \( f_{\text{visc}} \) is proportional to the relative velocity between bubbles, \( \Delta v \). The amount of energy dissipated per pair of sliding bubbles in a time interval \( \Delta t \) is \( (f_{\text{visc}} \times \Delta v) \times \Delta t \), which scales as \( \Delta v^2 \Delta t \). The crucial observation is that the energy fed into the system by shear needs to be balanced by the averaged amount of energy dissipated by local bubble sliding:

\[
\sigma \dot{\gamma} \sim \langle \Delta v^2 \rangle.
\] (4)

This simple relation directly links the rheology and fluctuation strength. Let us first imagine that our foam behaves as a Newtonian fluid, for which \( \sigma = \eta \dot{\gamma} \) with steady viscosity \( \eta \). In this case the steady state power balance equation, eqn (4), requires that \( \eta \dot{\gamma}^2 \sim \langle \Delta v^2 \rangle \), so that \( \Delta v \) and \( \dot{\gamma} \) scale in the same way. However, foams above \( \phi_c \) do not behave as Newtonian fluids, particularly for slow flows. Let us assume that the stress varies as a power law of the strain rate, which is a good approximation of a typical Herschel-Bulkley rheology at high flow rate: \( \sigma \sim \dot{\gamma}^\beta \).

Substituting this into the steady state power balance equation, eqn (4), one obtains \( \dot{\gamma}^{1+\beta} \sim \langle \Delta v^2 \rangle \), so that \( \Delta v \) and \( \dot{\gamma} \) do not scale in the same way; rather \( \Delta v \sim \dot{\gamma}^{(1+\beta)/2} \). For the typical case that \( \beta < 1 \), one concludes that the velocity fluctuations decay sub-linearly with the strain rate. In other words, the relative velocity fluctuations \( \langle \Delta v \rangle / \dot{\gamma} \) diverge as \( \dot{\gamma}^{\beta-1/2} \). This divergence is even stronger for flows slow enough that the stress reaches a plateau value: \( \sigma = \sigma_{\text{y}} \). The steady state power balance equation, eqn (4), then predicts \( \Delta v / \dot{\gamma} \) to diverge as \( \dot{\gamma}^{-1/2} \).

Similar balance arguments can also be used to rationalize the rheological data shown in Fig. 4e–f, where only in the disordered foams a yield stress was observed. This can be understood by noting that the energy balance argument shows that for flows in the regime where the stress plateaus, the relative fluctuations must diverge when the strain rate goes to zero. Since the fluctuations are constrained in the ordered system, they cannot diverge, energy cannot be dissipated strongly enough, and as a result there cannot be a yield stress—consistent with the data in Fig. 4e. In the disordered system, the fluctuations are not constrained, and nothing forbids the emergence of a finite yield stress.

We stress here that the role of disorder is to facilitate the nontrivial role of the fluctuations. For the case of elasticity of disordered media, disordered bubble motion is intimately connected to anomalous scaling of the shear modulus. For the flow of disordered media, nontrivial fluctuations enhance dissipation and lead to anomalous scaling of the stress–strain rate relation.

4. Scaling relations for flow near jamming

The single balance equation (4) is not sufficient to predict both \( \beta \) and the fluctuations—we need additional arguments to obtain a set of closed equations. For simplicity, below we first fix the elastic (\( \alpha_1 \)) and viscous exponents (\( \alpha_{\text{visc}} \)) to unity.

We now summarize a recent scaling model that provides the needed closure relations. The model has three ingredients.

1. The system is in power balance.
2. Flowing foam can be mapped to a static system characterized by an effective strain \( \gamma_{\text{eff}} \).
3. The stress as a function of \( \gamma_{\text{eff}} \) is given by an elastic constitutive relation.

In the limit of very wet foams, i.e., \( \phi \downarrow \phi_c \), the model is particularly simple. Once the foam is flowing there will be strong fluctuations that introduce a nontrivial strain scale into the problem: \( \gamma_{\text{dyn}} = \gamma / \Delta v \), where \( d \) is a typical bubble diameter. This strain can be thought of as characterizing the global deformation where locally all bubbles have rearranged, and we now assume that \( \gamma_{\text{eff}} = \gamma_{\text{dyn}} \). For the constitutive equation, we build on recent work that shows that near jamming, the linear shear modulus vanishes and the stress scales as \( \sigma \sim |\gamma|^{1/2} \).

We refer to this regime as the critical regime, and here we predict the rheology of the form \( \sigma \sim \dot{\gamma}^{1/2} \), and a divergence of the relative fluctuations \( \Delta v / \dot{\gamma} \sim \dot{\gamma}^{-1/4} \) which is in very good agreement with numerical simulations of the bubble model.

In the limit of very slow flows and for \( \phi > \phi_c \), the model also becomes very simple. In that regime, the effective strain is no longer expected to be dominated by dynamic effects, and we postulate that the effective strain in the system then equals \( \Delta \phi \equiv \phi - \phi_c \). For the equation for the stress, recent work shows that above jamming the shear modulus \( G \) scales with the distance to the jamming point as \( G \sim \Delta z \sim |\Delta \phi|^{1/2} \). In summary

\[
\begin{align*}
\sigma_{\text{eff}} \sim \Delta v^2 & \quad \Rightarrow \quad \Delta v \sim \Delta \phi^{3/4}\dot{\gamma}^{1/2} \\
\gamma_{\text{eff}} \sim \dot{\gamma} \Delta v & \quad \Rightarrow \quad \gamma_{\text{eff}} \sim \Delta \phi^{1/4} \\
\sigma_{\text{eff}} \sim \Delta \phi^{1/2} \gamma_{\text{eff}} & \quad \Rightarrow \quad \sigma_{\text{eff}} \sim \Delta \phi^{3/2} \\
\end{align*}
\] (6)

So in the yield stress regime, the model predicts the yield stress \( \sigma \sim \Delta \phi^{3/2} \) and an even stronger divergence of the relative fluctuations.

The general form of the model is obtained by combining these regimes, so that \( \gamma_{\text{eff}} = a_1 \Delta \phi + a_2 \dot{\gamma} / \Delta v \), and \( \sigma = a_3 |\Delta \phi|^{3/4} \gamma_{\text{eff}} + a_4 \gamma_{\text{eff}}^{1/2} \), where \( a_i \) are numerical constants to be determined.

It turns out that there is a third regime, which we refer to as the transition regime; here the three equations are

...
The behavior in the transition regimes interpolates in between the purely density dependent behavior of the yield regime and the purely strain rate dependent behavior of the critical regime. It can be shown that the yield stress regime dominates for \( \dot{\gamma} \leq \Delta \phi^{3/2} \), the critical regime for \( \dot{\gamma} \geq \Delta \phi^{3} \) and the transition regime in the range in between—which can span arbitrarily many decades when \( \Delta \phi \) tends to zero.

The flow curves predicted by eqn (5)–(7) are indeed consistent with experiment—see, e.g., the results of Nordstrom et al. for dense suspensions of soft colloidal particles. We note also that the transition regime implies deviations from Herschel–Bulkey phenomenology. These are difficult to observe but consistent with our observation that very often, Herschel–Bulkey-fits to experimental or numerical data underestimate the data in the crossover regime between yield stress and power law behavior—precisely the regime where the transition rheology is predicted.

**General microscopics**

What happens when the microscopic exponents (\( \alpha_{\text{el}} \)) and (\( \alpha_{\text{visc}} \)) are unequal to one? It is easy to see that the microscopic exponents influence the connection between fluctuations and rheology: assuming that the macroscopic rheology is of the form \( \sigma \propto \dot{\gamma}^{\alpha} \), the power balance equation then reads \( \dot{\gamma}^{\alpha} \sim (\Delta \nu^{1-\alpha}) \). This shows that details matter in the sense that different local dissipative laws directly affect the precise form of the balance equation (4)—in this sense, the physics is not universal. But the mechanism linking fluctuations and rheology is.

Delving into more details, for the yield stress regime, the stress–strain relation is affected by the value of \( \alpha_{\text{el}} \), as it sets the scaling of the shear modulus \( G \) with \( \Delta \phi \). Prior work has shown that for nonlinear interactions, the stress–strain relation becomes \( \sigma \sim \Delta \phi^{1/2} \dot{\gamma} \). For the critical regime, both the elastic and viscous exponents matter. Let us keep the elastic exponent equal to one, which is a good approximation for foams. Recall that \( \beta \) should, in general, depend on \( \alpha_{\text{visc}} \). Repeating the analysis in the critical regime for arbitrary \( \alpha_{\text{visc}} \) yields

\[
\beta = \frac{2\alpha_{\text{visc}}}{\alpha_{\text{visc}} + 3}
\]

For \( \alpha_{\text{visc}} = 2/3 \), as is the case for mobile surfactants, we obtain \( \beta = 4/11 \approx 0.36 \), in remarkably good agreement with the value \( \beta \approx 0.36 \) found in experiments.

Finally, we point out here that for dense granular media, the main dissipation comes from sliding friction, which could be seen as a viscous interaction with exponent zero—the frictional forces do not change appreciably with the sliding rate. In granular media it has been taken as a triviality that both the local and global interactions are very similar, namely frictional. We doubt that granular flows are affine, but we believe that this correspondence between local and global flow behavior is a lucky coincidence—we note here that for \( \alpha_{\text{visc}} = 0 \), the global flow exponent \( \beta \) also becomes zero in the scaling model.

5. Oscillatory rheology

In many cases oscillatory rheology is the preferred experimental tool to capture the general viscoelastic behavior of soft materials. Oscillatory rheology within the bubble model displays critical scaling near \( \phi_{c} \), as we now discuss—yet again, non-affine fluctuations play a key role.

When a viscoelastic material is driven with a small amplitude strain \( \gamma(t) = \gamma_{0} \sin \omega t \), the steady state stress is also sinusoidal and has both in-phase (elastic storage) and out-of-phase (viscous loss) contributions, \( \sigma(t) = G'(\omega)\sin \omega t + G''(\omega) \cos \omega t \). Together the amplitudes of these two contributions define the frequency dependent complex shear modulus \( G^* = G' + iG'' \). In a slowly sheared viscoelastic solid, the storage modulus \( G' \) approaches the quasistatic shear modulus \( G_{0} \). At the same time the loss modulus \( G'' = \eta_{0}\omega \) is linear in frequency, i.e. Newtonian; the prefactor \( \eta_{0} \) is known as the dynamic (versus steady) viscosity.

As we noted above, the shear modulus of weakly jammed soft spheres is determined by the material’s proximity to isostaticity, \( G_{0} \sim \Delta \sim \Delta \phi^{1/2} \). How does the dynamic viscosity scale? The viscosity governs the power dissipated in each cycle; at low
frequencies this is written as $P = (1/2)\eta_0b^2\gamma_0^2$. Dissipation at the bubble scale is dominated by the bubbles’ tendency to slide past their neighbors, thereby avoiding the energetic penalty of interpenetration and generating strongly non-affine motion. A balance argument reveals that the quasistatic sliding amplitude scales as $\Delta u^0 \sim \gamma/\Delta z^{1/2}$. Then, taking $\omega_{\text{visc}} = 1$, the dissipated power is dominated by transverse sliding, $P \sim \int \omega_{\text{visc}} \Delta \tilde{v} \sim (\Delta u^0 \omega_{\text{visc}})^2$, and the dynamic viscosity $\eta_0 \sim 1/\Delta z$ grows dramatically near unjamming.

Oscillatory rheology allows us to probe the breakdown of quasistatic elastic response by increasing the driving frequency. Surprisingly, this breakdown and the emergent rate-dependent quasistatic elastic response by increasing the driving frequency can be predicted from quasistatic scaling relations. This is most easily seen via a critical scaling ansatz; its results are verified by more rigorous treatments. One simply assumes that $G^*$ obeys one parameter scaling with a characteristic time scale $\tau^* \equiv \eta_0/G_0 \sim 1/\Delta z^2$ determined by the quasistatic moduli:

$$G^*(\omega, \Delta z) = \bar{G}^* (\omega \tau^*).$$  \hspace{1cm} (9)

By requiring continuity of the dimensionless scaling function $\bar{G}^*(x)$ and finite response at $z = z_c$, one finds that $\bar{G}^*$ must scale as $(ix)^{1/2}$ when $x \gg 1$. In other words, both the storage and loss moduli scale anomalously as the square root of frequency when the system is driven faster than $1/\tau^*$. Numerics confirm this prediction (Fig. 6a), again, with steady flow, macroscopic response differs qualitatively from the local forces. Experimental systems do indeed display anomalous power law scaling in their loss moduli – see Fig. 6b.

6. Conclusion and outlook

In this paper we have proposed that static and dynamic properties of foams can be captured by recent ideas stemming from jamming – and we have also illustrated how some of these theoretical ideas grew out of experimental questions (“how does foam lose rigidity”) and observations (“fluctuations in foams seem to grow”).

We suggest two broad directions for future research. On the one hand, there is a general lack in experimental data confronting some of the predictions, for example in the degree of nonaffinity as a function of wetness or the distribution of relative velocity fluctuations as a function of strain rate; more experiments are called for. In particular there are only a handful of studies of the mechanics and flow of very wet foams or emulsions – the recent advent of combined 3D confocal imaging and rheology of emulsions should open new experimental avenues to probe the critical regime.

On the other hand, many of the commonly observed phenomena in soft materials are without a sound description. What is the physics of plastic rearrangements? What is the fate of topological rearrangements (“T1 events”) when the foam gets increasingly wet? What about memory effects and reversibility? We look forward to many more surprises in the rich physics of collections of bubbles and droplets.

References


