Brewster cross polarization

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When a beam of light impinges upon a plane interface separating two transparent media, it produces reflected and transmitted beams. In 1815 the Scottish physicist Brewster [1] discovered the total polarization of the reflected beam at the angle $\theta_B$ since named after him. From his observations he was also able to empirically determine the celebrated equation known as Brewster’s law, $\tan \theta_B = n_1/n_2$, where $n_1$ and $n_2$ are the respective refractive indices of the two media. Several articles have been published on this subject. Fainman and Ko˝házi-Kis [3] theoretically derived and experimentally confirmed cross-polarization effects occurring at Brewster incidence. Shortly afterward, Li and Vernon [4] addressed the same problem using a microwave Gaussian beam; they found a cross-polarized component. More recently, Nasalski and Pagani [6] in a series of interesting papers. However, these studies fail to compare the intrinsic XPC owing to the natural angular spread (namely, the focusing) of the incident beam and the nonintrinsic one caused by reflection of such a beam at a dielectric interface. The main aim of this Letter is to fill this gap.

The structure of this Letter is as follows. We first solve the general problem of the reflection of a polarized fundamental Gaussian beam at the plane interface between two optical media [7,8]. Next, we derive analytical expressions for the polarization-dependent transverse spatial profiles of the reflected beam. From this result, we are able to prove that nonintrinsic XPC scales quadratically with the angular spread $\theta_0$ of the incident beam as opposed to the intrinsic XPC that scales as the fourth power of $\theta_0$. Finally, we present experimental confirmations of our theoretical findings.

Consider a monochromatic beam of light incident upon a plane interface that separates air from glass. With $n = n_{\text{air}}/n_{\text{glass}}$ we denote the ratio between the two refractive indices. As the beam meets the interface coming from the air side, it will be convenient to take the axis $z$ of the laboratory Cartesian frame $K=(O,x,y,z)$ normal to the interface and directed from the air to the glass. Moreover, we choose the origin $O$ in a manner that the plane interface has equation $z = 0$. The air–glass interface and the incident and the reflected beams are pictorially illustrated in Fig. 1. In addition to the laboratory frame, we use a Cartesian frame $K_i=(O,x_i,y_i,z_i)$ attached to the incident beam and another one $K_r=(O,x_r,y_r,z_r)$ attached to the reflected beam. Let $\mathbf{k}_0=\mathbf{e}_z k_0$ and $\mathbf{k}$ denote the central and the noncentral wave vectors of the incident beam, respectively, with $|\mathbf{k}| = |\mathbf{k}_0| = k_0$. Then, the electric field of the incident beam can be written as a linear superposition of the fundamental vector plane-wave mode functions $\mathbf{\hat{e}}_i(\mathbf{k}) = \mathbf{e}_i(\mathbf{k}) \exp(i \mathbf{k} \cdot \mathbf{r})$ with complex amplitudes $a_i(\mathbf{k})$ as follows:

$$E^i(\mathbf{r}) = \sum_{\lambda=1}^{2} \int a_{i\lambda}(\mathbf{k}) \mathbf{\hat{e}}_{i\lambda}(\mathbf{k}) d^2k_T, \tag{1}$$

where $\mathbf{k}_T = \mathbf{k} - \mathbf{k}_0(\mathbf{e}_0 \cdot \mathbf{k})/k_0^2$ is the transverse part of $\mathbf{k}$, and we choose the polarization unit basis vectors as $\mathbf{e}_{i\lambda}(\mathbf{k}) = \mathbf{e}_{i\lambda}(\mathbf{k}) \times \mathbf{k}/k_0$ and $\mathbf{e}_{i\lambda}(\mathbf{k}) = \mathbf{z} \times \mathbf{k}/|\mathbf{z} \times \mathbf{k}|$ [9]. Here $\mathbf{z}$ is a real unit vector directed along the laboratory axis $z$. By definition, $\mathbf{e}_i(\mathbf{k})$ lies in the plane of incidence containing both $\mathbf{k}$ and $\mathbf{z}$ while $\mathbf{e}_d(\mathbf{k})$ is orthogonal to such a plane. A plane wave whose electric field vector is parallel to either $\mathbf{e}_1(\mathbf{k})$ or $\mathbf{e}_2(\mathbf{k})$ is referred to

Fig. 1. (Color online) Geometry of beam reflection at the air-medium interface; $\theta_B$ is the Brewster angle.
as either a TM or a TE wave, respectively. The symbols $S$ for TE and $P$ for TM, are also widely used. In Eq. (1) $a_\lambda(k) = A(k) \alpha_\lambda(k)$, where $A(k)$ and $\alpha_\lambda(k)$ are the scalar and the vector spectral amplitudes of the field, respectively. Here we consider a monochromatic Gaussian beam whose spectral amplitude $A(k)$ is localized in $k$ space, centered at the central wave vector $k_0 = k_0 \hat{z}_0$, on the surface of equation $\omega^2(k) = c^2 k_0^2$, namely,

$$A(k) = e^{-[|k_0|^2 k^2]^{1/2} / 2} / (1 - |k_0|^2)^{1/2},$$

where $\theta_0 = 2/(k_0 w_0)$ is the diffraction-defined angular aperture of the incident beam [10] that has, by hypothesis, a minimum diameter (spot size) equal to $2w_0$ located at $z_i = -d$. The vector spectral amplitudes are defined as $\alpha_\lambda(k) = \hat{e}_\lambda(k) \cdot \hat{f}$, with $k^2 \hat{f} = (f_p \hat{x} + f_s \hat{y})$, and is a complex-valued unit vector that fixes the polarization of the reflected beam.

When the latter is reflected at the interface, each vector mode function changes according to

$$\hat{\chi}_\lambda(k) \rightarrow r_{\lambda}(k)\hat{\chi}_\lambda(k),$$

where $r_1(k)$ and $r_2(k)$ are the Fresnel reflection amplitudes for TM and TE waves, respectively [11], and $\hat{\chi}_\lambda(k) = k\hat{z}$ is set by the law of specular reflection [12]. If we substitute Eq. (3) into Eq. (1), we obtain

$$E'(r) = \sum_{\lambda=1}^{2} a_\lambda(r)\rho_\lambda(k)\hat{\chi}_\lambda(k)d^2k_T,$$

where $\rho_0 = k_0 \hat{z}_0$, by definition. The expression for the magnetic field $B^R(r)$ of the reflected beam may be obtained from the equation above via the straightforward substitutions $a_\lambda(k) \rightarrow b_\lambda(k)/c$, where $b_1(k) = -a_2(k)$, $\rho_1(k)/\rho_2(k)$ and $\rho_2(k) = a_1(k) \rho_1(k)/\rho_2(k)$.

From Eq. (2) it follows that $A(k) = 0$ for those wave vectors $k$ lying outside the paraxial domain $P = \{k : k^2 |k_0|^2 \leq \theta_0^2\}$, with $\theta_0 \ll 1$ for well-collimated beams. This allows us to calculate analytically $E^R$ (and, similarly, $B^R$) via a power series expansion for the integrand of Eq. (4) about the point $k = k_0$ up to and including second-order terms in $k_0$.

In practice, we extend to the problem at hand the perturbative approach introduced by Lax et al. [13] and further developed by Deutsch and Garrison [14]. It is easy to see that if $\gamma(k) = \arccos(k \cdot k_0/|k_0|^2)$ we denote the angle between the central wave vector $k_0$ and the noncentral one $k$, then we can write $k_{\parallel} / k_0 = \sin \gamma(k) / \gamma(k)$, where $\gamma(k) \leq \theta_0 \ll 1$, with $\theta_0$ the natural small parameter for the power series expansion [14]. Explicit expressions for the power series expansions of both $E^R(r)$ and $B^R(r)$ are given in Appendix A in [15].

With the knowledge of both $E^R(r)$ and $B^R(r)$ it is possible to calculate the intensity spatial distribution (i.e., the beam profile) $I(r)$ of the reflected beam as the flux of the cycle-averaged Poynting vector $\vec{S} \approx \text{Re}(\vec{E} \times \vec{B}^*)$ across a surface perpendicular to the central direction of propagation $\hat{z}_0$, namely,

$$I(r) = \vec{P} \cdot \hat{z}_0 = \text{Re}(E^R \times B^R) - \text{Re}(E^S \times B^S),$$

where $E^S(r) = \text{Re}(-E^R \times B^R)$ are the intensity distributions produced by the $P$- and $S$-polarized components of the reflected beam, respectively. Here $P$ and $S$ polarization directions are defined with respect to the central plane of incidence containing $\hat{z}$, $k_0$, and $\hat{r}_0$. After a lengthy but straightforward calculation it is not difficult to obtain, for a $P$-polarized incident beam (i.e., for the choice $f_p = 1$ and $f_s = 0$), the following power series expansions:

$$I_P(r) = I_0(r) = r^2 + \theta_0 uX + \theta_0^2 / (v + pX^2 + qY^2),$$

$$I_S(r) = I_0(r) = \theta_0^2 Y^2,$$

where $X = x_0 / w_0$, $Y = y_0 / w_0$, $Z = (x_0 + d) / L$, and

$$I_0(r) = \text{exp}[-2 / (X^2 + Y^2) / (1 + Z^2)]$$

is the intensity distribution of the incident beam. In Eqs. (5) and (6), $r_P$ and $r_S$ are the Fresnel reflection amplitudes for $P$ and $S$ waves, respectively, evaluated at the central angle of incidence $\theta = \text{arccos}(k_0 \cdot \hat{z}_0 / k_0)$ while $u, v, p, q, s$ are some complicated functions of $Z, \theta, r_P, r_S$, and their derivatives, whose explicit form is given in Appendix B in [15]. At the Brewster angle of incidence $\theta = \theta_B = \text{arctan}(n)$, only $p$ and $s$ take a nonzero value, namely,

$$p(1 + Z^2) = \rho_d \rho_d \psi_0^2, \quad s(1 + Z^2) = r^2 / n^2 |\psi_0|^2,$$

and Eqs. (5) and (6) reduce to

$$I_P(r) = I_0(r) = \theta_0^2 Y^2, \quad I_S(r) = I_0(r) = \theta_0^2 Y^2,$$

respectively. From this result it immediately follows that the ratio $\rho$ between the power of the $S$ and the $P$ components of the reflected beam is Brewster incidence is simply equal to $s/p$,

$$\rho = \left( \frac{n^2}{\rho_d \rho_d \psi_0^2} \right)^2.$$

This simple result is remarkable, since it shows that $\rho$ is independent from the waist $w_0$ of the incident beam, that is, $\rho$ take the same value for either a well-collimated or a strongly focused beam.

Equations (5) and (6) represent the main theoretical result of this Letter. In particular, Eq. (6) shows that nonintrinsic XPC generates an intensity $I_0(r)$ that scales quadratically with $\theta_0$. This behavior cannot be ascribed to the intrinsic XPC exhibited by the incident beam, since the latter is due to the beam-divergence only and the consequent cross-polarized intensity scales with $\theta_0^2$ [16,17]. Moreover, Eq. (8) shows two important things. First, we see that even at a Brewster angle of incidence the extinction of a
P-polarized beam is not perfect as \( (\text{dr}_P/\text{d}\theta)|_{\theta_P} \neq 0 \) and \( \text{r}_S|_{\theta_S} \neq 0 \). Thus, although the input beam is \( P \) polarized, an \( S \) component appears after reflection. Second, the two expressions in Eq. (8) show that after reflection, the cylindrical symmetry about the axis of propagation of the beam is lost and two orthogonally polarized TEM\(_{10} \) and TEM\(_{01} \) modes are generated. The fact that \( I_S \) has a TEM\(_{01} \) profile, as opposed to the cloverleaf TEM\(_{11} \) pattern typical of the cross-polarization intensity of the incident beam, is consistent with the hypothesis that such a term originates from nonintrinsic XPC and is not a simple beam-divergence effect.

We verified these theoretical results in our laboratory by using a super-luminescent light-emitting diode (SLED) operating at \( \lambda=820 \) nm (InPhenix IP-SDD0802) as a light source. The output of the SLED was first spatially filtered by a single-mode optical fiber to prepare the input beam into the fundamental mode (SLED) operating at \( \lambda=820 \) nm (InPhenix IP-SDD0802) as a light source. The output of the SLED was first spatially filtered by a single-mode optical fiber to prepare the input beam into the fundamental Gaussian mode and then collimated by a microscope objective to produce a very large beam waist \( (w_0=1.64 \text{ mm}) \) before passing across a polarizer selecting \( P \) polarization. A second lens put behind the polarizer generated the desired waist for the input beam. The so-prepared beam was sent upon the surface of a right-angle BK7 glass \( (n=1.51) \) prism mounted onto a precision rotation stage with a resolution of \( 9 \times 10^{-6} \) rad (Newport URS-BCC) to accurately determine the Brewster angle \( (\theta_B=56.49^\circ) \). Finally, the polarization-dependent beam intensity profiles after reflection were recorded by a CCD-based beam intensity profiler (Spiricon LBA-FW-SCOR-20) mounted behind a polarizer put along the axis of the reflected beam at a large distance from the interface (far-field measurement). A qualitative comparison between calculated and measured intensity distributions is shown in Fig. 2. The measured ratio \( \rho_{\text{exp}} \) of the \( S \)-polarization component power to the \( P \)-polarization one at Brewster incidence was \( \rho_{\text{exp}}=0.20\pm0.05 \), in excellent agreement with the theoretical prediction of Eq. (9) giving, for BK7 glass, \( \rho_{\text{th}}=4n^4/(1+n^2)^4 \approx 0.18 \).

In conclusion, we found that when a TM-polarized fundamental Gaussian beam is reflected at Brewster incidence it generates a two-mode beam with both a dominant and a cross-polarized component. The intensity of the latter scales quadratically with the angular divergence of the incident beam and can be, therefore, orders of magnitude bigger than the intrinsic cross-polarized intensity of the incident beam that scales with the fourth power of \( \theta_B \).

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**References and Notes**

9. Throughout this Letter we use the symbols “·” and “×” to denote the ordinary scalar and vector products in \( \mathbb{R}^3 \), respectively.

![Fig. 2.](image-url) (Color online) Calculated and measured intensity transverse spatial profiles of the \( P \)- and the \( S \)-polarized modes of the reflected beam. The beam waist of the incident beam was \( w_0=34 \) \( \mu \text{m} \).